

# Teaching and Learning perspectives on Numbers Systems within a first year tertiary IT course

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## ABSTRACT

In this paper the author investigates which teaching and learning techniques are effective in helping first-year IT students grasp number system concepts. The culture, academic background and prior mathematical experience are factors in how students choose to learn these skills. The teaching and learning principles which underpin effective pedagogy are discussed and teaching materials are presented which are of immediate relevance to students studying computer networking, hardware and programming. The author provides a lesson plan illustrating a detailed, visual and yet flexible constructivist approach to teaching conversions between decimal and binary systems to help students gain fluency in conversion. The author concludes that making explicit connections between prior number knowledge, being flexible in which tool or device is used to perform calculation and providing a contextual focus helps students learn this skill quickly and effectively. The conversion skills then can be easily adapted to octal and hexadecimal systems as required.

**Keywords:** Mathematics, Number Systems, Binary, Numeracy, group work

## 1. INTRODUCTION

The everyday world has numbers, numeracy and mathematics woven into its fabric. Numeracy is typically hidden within workplace, home and family contexts and is usually associated with informal learning. In tertiary education contexts, numeracy is highly visible and the specific knowledge, skills and competencies required of learners will depend upon many factors. The discipline being studied, for example Information Technology or Computer Science, the needs of the employers within the field, the academic level of the programme, the existing subject knowledge learners bring to their study environment, the contextualisation of the teaching and the theories of learning which influence how the teaching materials are developed and presented, are some of these factors. Coben (2000, p.35) defines being numerate as being “competent, confident, and comfortable with one’s own judgements on whether to use mathematics in a particular situation, and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context.” The use of mathematics in specific situations, for example first-year IT students studying network addressing, highlights the importance of context in the teaching and learning of numeracy. Lord, Hart and Springate (2010, p. 211) citing 12 sources, state that “The need to contextualise learning and ensure it relates to the skill requirements of the workplace was the factor which resonated most strongly in the literature”. The importance of contextualising the learning is clearly signalled and has implications for the teaching and learning of numeracy within adult education, thus teaching number systems to first-year IT students should occur in the context of a networking course, where it has immediate relevance. The teacher needs to have a sound appreciation of conceptions of numeracy and teach the use of mathematical skills for a particular purpose. Students will be more motivated to persist in problem-solving when the purpose and relevance of the learning tasks is explicit. What factors and teaching approaches are important when teaching number systems to first year IT students? This paper aims to address this question and suggest practical techniques to

facilitate this process.

## 2. BACKGROUND AND LEARNER CONTEXT

The context in which the learners are taught mathematical skills is part of lessons presented to first year degree Information Technology (IT) students completing an introductory course in computer networks. Students are required to develop conceptual and practical foundations in number conversion between binary and decimal systems as part of understanding numerical computer addressing schemes. The skills, knowledge and concepts developed at this level provide the basis for the second year networking course. Tertiary and vendor-specific courses require competence with the binary and hexadecimal number systems as they apply to specific computer software and hardware address schemes. Binary number systems also form the basis for digital electronic logic circuitry found within computers and supports propositional logic studies in computer science and in Human-Computer Interaction courses.

The learners come from a variety of educational backgrounds, mostly with English as their first language meeting basic literacy and numeracy requirements through an application process. The majority of the learners are New Zealand Pakeha, some are Maori and Pacific Island students, and a minority are Asian students. Some students have come directly from school and so have recent educational experience with mathematics at senior secondary level. Other students have had less recent formal experience with mathematics at school or in the workplace and are returning to study as part of a career change. All the learners are assumed to be able to perform arithmetic mentally, use calculators, spreadsheets or online calculation programs to arrive at an answer to the number conversions. An understanding of powers (exponents) is also assumed. The teaching of specific calculator model use and spreadsheet functions would form part of separate teaching sessions supporting this topic.

## 3. TEACHING AND LEARNING PRINCIPLES

Teaching adults mathematical skills as part of developing numerate behaviour will be influenced by many factors such as the teacher’s view of numeracy, subject knowledge, the teacher’s familiarity with research in the field, the class

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composition and the learning environment. The distinction between numeracy and mathematics is also the subject of ongoing debate. Manly, Tout, van Groenestijn, and Clermont, (2001, p. 79) state that “ Our basic premise is that numeracy is the bridge that links mathematical knowledge, whether acquired via formal or informal learning, with functional and information-processing demands encountered in the real world.” Numeracy seen from this perspective implies a sense of knowledge construction, by valuing existing mathematical knowledge, and linking this to contextualised application in real life situations. Establishing what is known by students about mathematics is not a trivial issue and requires that the teacher gains a holistic knowledge base about each student. Aspects that have been identified as important in adult numeracy teaching approaches include the role of group work, discussion, higher order thinking questions, making connections between topics and a positive teacher attitude to the subject.

Learning environments, whether face to face or online have a social element where students tend to group together when focussed on a common task or wish to share their knowledge with each other. The phenomenon and success of social networking websites is a visible testament to the natural sense of camaraderie students have inside, outside, and during class. The value of group work, especially small group work, is well known within the education field. Latu (2004) investigated Maori and Pacific Island senior school students’ participation in a University summer scholarship programme. The study revealed that students expressed positive preferences for group work in a collaborative and informal atmosphere. The supportive nature of such groups moderated the feelings associated with failure, especially for low-achieving students. In another study of teaching numeracy to Maori school students, Higgins, Parangi, Wilson and Klaracich (2004) found that group work was highly effective in numeracy teaching, where the class was viewed more as a family unit of interconnected groups, where Tuakana Teina, which refers to the relationship between an older (tuakana) and younger (teina) person, especially in the role of peers, enabled older and younger to learn from each other. The range of ages within the class can be used to benefit knowledge sharing during group work and class work. The responsibility of the students to look after each other’s learning and to collectively master the numeracy tasks stood out strongly. Although results from such studies may differ if conducted with a more diverse student demographic, the benefits of group work apply closely to most learning contexts. The importance of talking about mathematics and extensive discussion was also highlighted in both of the above studies. However, group work can be problematic for students’ learning where dominant group members may not facilitate fair contributions from all involved. Mousley and Campbell (2007) study on mathematics education students found that group work, especially group assessment, did not easily translate into effective collaboration and good quality discussion. Group work is clearly of value in effective numeracy teaching but teacher management and guidance of the groups has to be a vital part of this approach. Collaboration typically requires discussion to occur.

Traditionally, mathematics teaching expects students to learn the necessary skills as demonstrated by the teacher, by attempting innumerable example problems alone, and in silence. The importance of students talking about what they are learning and while they are learning has become foregrounded in more recent years within educational contexts. Swain and Swan (2009) explored the efforts of teachers to blend research-based principles into numeracy teaching of post-16 year learners in an English context. Discussion arose

as one important approach to teaching mathematical concepts. Using extensive talk about students’ solutions to problems can uncover incorrect conceptions formed during tasks and can help students show their reasoning. Discussion is also a valuable means of helping students learn to justify their approaches to others, especially peers. Muir (2008) studied the relationship between teacher’s actions and principles of practice in a school context. Discussion stood out as a key approach supporting other valuable principles identified in this research. Open discussion generates multiple perspectives which can expand the understanding of concepts by providing shared insights. Discussion frequently involves answering questions posed by the teacher and by the students as a result of their group work. The depth of the questions posed can stimulate various levels of thought.

Questioning as an approach to teaching has a long history and is widely used by both teachers and learners to progress learning sessions. Swain and Swan (2009) findings strongly support the use of probing questions as a means to foster high-order thinking in students. The use of such approaches can promote deeper topic exploration than would be achieved through recall alone. Hodgen, Coben and Rhodes (2009) citing Black and William’s extensive literature review of innovations supporting frequent feedback to students about their learning, highlight questions that stimulate learners to think rather than verbalise recall. However, question construction and delivery is a complex task which challenges numeracy teaching. Swan (2005, p. 32) identifies several common pitfalls including “..asking questions with no apparent purpose; asking too many questions; asking several questions at once; poor sentencing; ignoring incorrect answers and not taking answers seriously..”. Questioning that promotes thinking can be a very powerful means of making connections between and within topics to enhance depth of insight.

Effective teaching of numeracy requires the teacher to help students make connections between topics to promote overall concept development. Muir (2008) identifies the importance of making explicit connections between topics through student group work, involving words, symbols and other visual representations. The use of discussion directed to a common goal can help students make the connections themselves, thus supporting motivation and problem resolution. Selecting appropriate examples for the teacher to make connections and highlight similarities requires careful attention to how closely the characteristics of the example match the concept being taught. Marr (1998, p. 15) emphasises the need for teachers to “.. concentrate teaching efforts on ensuring we create connections for the students.” This typically involves revisiting concepts, trying to think about them from other useful perspectives and actively engaging students in this task. However, teachers must not be too quick to jump to instant or disparate connections without progression. Anthony and Walshaw (2009) note that teachers must be careful to sequence material and advocate progressive modifications to the student’s existing knowledge base when making links between topics coherently. The attitude that the teacher has to the subject, and the personal teaching style adopted when making connections, may enhance or inhibit effective teaching and learning.

#### 4. TEACHING MATERIALS

Adult numeracy teaching and learning contexts vary widely in terms of their physical environments, availability of resources, the collective knowledge of the learners, the teacher’s subject content knowledge, the teacher’s teaching style, and how these components are organised for the benefit of the students. The teacher’s views and beliefs about numeracy and how

adults learn also influences the selection of teaching materials employed. The context is a tertiary level class of adult learners with varying abilities and prior knowledge, but a common motivation to study Information Technology (IT) at this level. The topic is central to understanding many allied computing concepts. The teaching materials selected for number conversion between decimal and binary are informed by research and teaching experience over several years. Traditional materials such as pen and paper complement the use of IT in assisting students, not only to be confident to calculate the number conversions correctly, but more importantly to develop conceptual insight about the logic of process.

The use of IT in education is now relatively ubiquitous, although resource availability remains a challenge in many contexts. Scientific calculator use, smart phone applications, computer applications, for example, Microsoft Excel, Scratch and Alice have been used successfully within secondary and tertiary settings. Carpentieri, Lister and Frumkin (2009) cite several reports which emphasise the importance of including IT in numeracy learning. The use of IT for education and entertainment in the home and work recognises the pervasive nature of technology in student's lives. Ginsburg (1998) observed that within adult education, IT is used as part of the curriculum; part of a content delivery mechanism, an instructional complement and as tool to instruct learners directly. A tertiary level IT course in computer networks uses technology in all of these ways. The impact of IT upon adult education is generally positive, but Hattie (1999) found that the size of this effect tends to reduce with increasing student age. The lesson forming part of this research makes effective use of IT. For example, a data projector is used during the session showing a decimal and binary number equivalents, the Windows calculator in scientific view is also shown in use on the projector and use of Microsoft Excel functions to convert between the number bases is shown. As part of a variety of teaching materials and strategies employed, pen and paper still has merit for use in number conversion lessons.

The ubiquity of pen and paper in the traditional mathematics classroom may have negative connotations for many learners, but their use is still of value for reasons of availability, creativity, tactile kinaesthetic appeal, communication of process, symbol use and active student engagement in learning. Tablet computers can also implement this form of interaction with technology. Abzug (2008) research about teaching number representation and the performance of arithmetic within computers to university students emphasises the importance of pen and paper in developing cognitive skills in mental calculations. The student is more likely to know if the calculation is correct or not, whether transcription or operator precedence error has occurred when pen and paper are used. The writing down of intermediate results on paper during calculation is still used widely by students and teachers, especially if command of the inbuilt IT device memory is limited. Zeleznik, Miller, Li and La Viola (2008, p. 21) in presenting MathPaper, a pen-based interactive technology to support mathematical teaching and learning, readily acknowledge "the nature of pencil and paper itself, which affords fluid, direct specification of 2D mathematical notations and supporting diagrammatic structures in arbitrary juxtapositions." Traditional pen and paper workings feature in the number conversion lesson. An example of this is use of the whole number division and remainder method for decimal to binary conversion. Pen and paper also facilitates multiple ways and shortcuts to speedier calculation. The process of number conversion from decimal to binary is written out in sequence on the whiteboard and talked through with the students as it progresses.

The calculation strategies that teachers and students use, typically involve arithmetical processes to arrive at an answer to the problem at hand. Mathematics makes use of symbols and patterns leading to visual representations in constructing meaning. Strictly linear, step by step approaches to mathematics calculations have their place, especially in algorithmic processes within computers. By contrast, people use a range of strategies to achieve the same answer. Frequently these approaches are non-linear, have a random trial and error component and involve visual mapping. West (2011) presents and analyses a range of multiplication strategies used by students to calculate answers to mathematical problems, highlighting the relative efficiency of alternative algorithms, when compared with traditional ones. Flexibility of approach to number conversion forms part of teaching session. Calculator use, online aids or paper and pencil, including whiteboard use are all part of the diversity within the teaching session. An example from the lesson (see Appendix A) could include starting with the column of largest value in an eight bit number; write the number 1 in the column, so that the decimal value remaining is equal to or less than the column selected. This process is repeated until the total of the columns appears to be correct visually. The zeroes can then be filled in the remaining positions. A final check for the correct answer is achieved by adding up the decimal values where the number 1 has been written, and cross checking this with the original decimal number. The variety of approaches to visualising numbers, arithmetic processes and comprehension of mathematical symbol use, leads us to consider wider concepts, rather than be routine focused only. Appendix B presents an example worksheet to teach Decimal to Binary Conversion and Appendix C presents an example worksheet to teach Binary to Decimal conversion, providing practice from both perspectives..

Emphasis on technological approaches to calculation and problem solving has featured strongly in the above considerations of teaching materials used for number conversion. The need for students to develop good reasoning skills in their application of concepts does not necessarily follow from reliance upon IT based teaching materials, however professionally created and presented by experts in the field. Garry (2010) investigated employing the Socratic Method as part of problem based learning in relation to Cisco's Certified Networking Associate (CCNA) curriculum to promote deeper learning. The focus of this approach is to guide students' thinking and to encourage critical thought around a topic. Worksheet questions form part of the lesson planning for the decimal and binary conversions. An example from the lesson could be questions about whether the conversion process between decimal and binary using the divide and remainder method works for other number bases, and if the number converted can be other than base ten. This would extend the student to test out other possibilities, and potentially see other connections e.g. three binary bits equivalent to one octal digit.

## 5. CONCLUSION

This paper addressed the question "What factors and teaching approaches are important when teaching number systems to first year IT students?" Several teaching principles have been presented and critically justified in the light of research. The teaching materials have also been subject to critical consideration and suitability for purpose in supporting these teaching principles. Group work facilitates talk and discussion about mathematical content with potential to generate high order questions that may be solved within the group or within the class. If the teacher allows sufficient time for subject related talk, there is increased potential for connections to be

made within and between topics. Potential for further conceptual thinking is also present by grouping of binary digits into bases which are powers of two e.g. octal and hexadecimal. Links with propositional logic could also be made thus developing more abstraction, a particularly relevant approach to problem solving in IT contexts. The lesson builds on the students' prior knowledge of base ten numbers in constructing new knowledge. The combination of group work with multiple approaches to number systems conversion, coupled with pragmatic use of a range of devices, can support meaningful study. Such study could also foster confidence in students and potentially inspire them to apply themselves in related fields such as application development and computer game programming.

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## APPENDIX A: LESSON PLAN (2 HOURS INCLUDING BREAKS)

Environment The room has tables with pads of paper, pens, pencils, calculators and a whiteboard for use by all. An Internet – connected computer, with a data projector (and a roll-down screen) is also present for use by all.

### Aims of lesson

After completion of this lesson the learners will be able to:

- Convert positive whole numbers, within a given range, from Decimal (Base Ten) to Binary (Base 2)
- Convert positive whole numbers, within a given range, from Binary (Base 2) to Decimal (Base 10)

### Lesson Plan

Introduce myself, the lesson and the aim (purpose, rationale) of the lesson: To teach how to convert whole numbers between base 10 and base 2 in order to understand the central importance of the binary number system in the computing field.

I write up a three-digit number, between 10 and 255, e.g. **175** on the board. Students to work in groups of at least 2, ideally 3 and no more than 4 to identify what they know about what seems familiar, e.g. how to express 175 as “one hundred and seventy five”, and what information about this number **is not shown**. Each group member is to take notes and collate them for discussion later in class. This is the prior knowledge activation component.

After group discussion, body language, and time suggests that feedback time is appropriate, invite each group to share with the class their findings, verbally or via use of a whiteboard. The teacher asks probing questions to check for knowledge and to stimulate wider thought. For example: How do we know which digit is the largest one/smallest one? How do we know what is the **smallest single digit** that we can use? What is the **largest single digit** we can use? How does this link in with a ‘base 10 system’? What determines the largest number represented? Terminology use is monitored through questioning and response. Key terms (vocabulary e.g. base, powers, and column ‘weights, digits, binary digits (“bits”) are written on the whiteboard by the teacher and learners.

The teacher then invites each group to share the results of their discussion with the class. Teacher facilitates discussion by prompting, listening, and asking: if the ideas discussed could be applied to other number systems?

Key concepts taught: (1) The largest single digit that can be used in any given number base is **one less than the base number** e.g. for base 10 the single largest digit is:  $10 - 1 = 9$

Concept (2) The largest number that can be expressed is determined by: (a) the range of digits used and (b) the number of columns used.

(3) The **leftmost column** has the *highest* decimal weight and the **rightmost column** has the *smallest* weight [always 1 in contexts we are using]

Return to example:  $175$  is  $(1 \times 100) + (7 \times 10) + (5 \times 1) = 100 + 70 + 5 = 175$  thus the largest column is worth 100, the next smallest one 10 and the smallest one 1. ‘Units, Tens and Hundreds’.

Each column can also be expressed thus: base <sup>power</sup> e.g.  $10^2 = 100$ ,  $10^1 = 10$  and  $10^0 = 1$  (prior knowledge skills at this level).

Thus the same example  $175$  is  $(1 \times 10^2) + (7 \times 10^1) + (5 \times 10^0) = 100 + 70 + 5 = 175$ . Teacher writes up a few more examples and checks for understanding through discussion.

Principles outlined are now shown to be true for base 2 (binary). E.g. base is 2 so largest single digit is  $2 - 1 = 1$ . Smallest is zero. Using the same logic about base <sup>power</sup> for each column we show the following table.

4	2	1	Decimal weight
$2^2$	$2^1$	$2^0$	Base Powers
0	0	0	Smallest Binary Number
1	1	1	Largest Binary Number

In this example we see that 111 in binary is  $(1 \times 4) + (1 \times 2) + (1 \times 1) = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 7$  Other examples used e.g. 001, 011, 101 show the progression between columns in the same way as we do with 10 base systems. N.B. ‘Ten’ or 10 in base 10 can be expressed as “one oh” in base ten. The need to correctly make use of talk to clarify meaning is modelled by the teacher verbally.

On the projector the Windows Calculator, in Scientific View is shown with the conversion screen showing. 111 is input and the DEC button is clicked confirming 7 in base 10 is correct. Other examples demonstrated binary <----> decimal equivalents.

Three methods for conversion are presented: (1) Division and remainder algorithm is also demonstrated for conversion from decimal to binary. (2) Subtraction with a running total approach. (3) Excel functions = DEC2BIN and =BIN2DEC used to show identical answers.

A formula (vocabulary introduced) dividend/divisor = quotient + remainder **using whole numbers only** E.g. start off with a small number 6 to convert into binary:

